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An Application of Mathematical Modeling to the Study of Ecological Systems

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Abstract

In this thesis, we incorporate spatial construction into different ecological/epidemiological frameworks by applying the fix model. Firstly, we consider two specific expenses of dispersal: (i) the period of time spent for migration; (ii) passings during the dispersal cycle. Along with the deferred logistic development, we propose a two-fix model as far as postpone differential equation with two consistent time delays. The expenses of dispersal, without anyone else, just influence the population sizes at equilibrium and may try and drive the populations to extinction. With oscillations induced by the postpone in logistic development, numerical models are provided to illustrate the impact of misfortune by dispersal. Besides, we concentrate on a predator-prey framework in a two-fix environment with indirect impact (dread) considered. While perceiving a risk from predators, a prey might answer by reducing its reproduction and decreasing or increasing (depending on the species) its mobility. The benefit of an anti-predation reaction is additionally included. We investigate the impact of anti-predation reaction on population dynamics by analyzing the model with a fixed reaction level and study the anti-predation strategies according to an evolutionary perspective by applying adaptive dynamics.

Thirdly, we investigate the present moment or transient dynamics of some SIR infectious disease models over a sketchy environment. Employing the estimations of reactivity of equilibrium and amplification rates previously utilized in nature to concentrate on the reaction of an ecological framework to perturbations to equilibrium, we dissect the impact of the dispersals/goes among patches and other disease-related boundaries on momentary dynamics of these spatially organized disease models. These differences with most existing chips away at modeling the dynamics of infectious disease which are just interested in long haul disease dynamics as far as the basic reproduction number.

Keywords: dispersal, population dynamics, patch model, costs of dispersal, time delay, predator-prey.

Introduction

Population dynamics is an important subject which has wide applications in regions like environment, microbiology, epidemiology, virology, immunology, etc. There are millions of species in this present reality. Some of them interact with one another, affecting the population development of each and every species involved. Among a wide range of interactions between species, predatorprey type is generally interesting and complicated. This is mainly a direct result of its ubiquity and richness on the practical side, and the difficulties in mathematics it brings in. Besides, and importantly, transmission mechanism of infectious diseases is likewise of this kind, adding more weight to its significance.

Among those species in this present reality, some are extremely mobile, and it has been widely concurred that the spatial dispersion is one of the main variables responsible for the biodiversity. Accordingly, it is of particular importance both in practice and mathematics to concentrate on population dynamics in spatially heterogeneous environments, particularly the population dynamics of predator-prey type interacting species including transmission dynamics of infectious diseases.

Mathematical modeling and mechanisms of ecological patterns

Feedback

Feedback refers to a form in which an object interacts with the environment. It is the process of returning the output of the system to the input and changing the input in a way that has a loop between them, thereby affecting the function of the system, which is ubiquitous in ecological systems. There are four types of pattern forming feedbacks in plant communities, involving the water motion and then redistributing the water:

- Infiltration feedback between the water and plant biomass, meaning the higher infiltration rate of surface water into the soil in vegetation patches and the higher plant density, the higher soil–water content, and thus the higher plant growth rates;
- Root-augmentation feedback between above-ground biomass and roots of vegetation;
- Uptake feedback between biomass and water, meaning the depletion of soil-water due to the water-uptake by plant's roots;
- Soil–water diffusion feedback. We will present two types of feedback named as soil–water diffusion feedback and plant–soil negative feedback on pattern formation hereinafter. We deliberated the role of soil–water diffusion feedback in terms of vegetation patterns formation.

This mechanism portrays the ability of plant roots to absorb water. Specifically, the consumption of water induces a reduction of the holard in the location of plants. Consequently, compared to the surrounding water, there is a concentration difference of water that can cause a directional flow of water. The stronger the water absorption capacity of plants, the greater the concentration difference. It was characterized as (W − βN) by Hardenberg [19], in which W and N also represent water and plant biomass, and β reflects intensity of soil–water feedback. We introduced this formula to revised Klausmeier model in non-dimensional zed form:

$$\frac{\partial n}{\partial t} = wn^2 - mn + \Delta n,$$
$$\frac{\partial w}{\partial t} = a - w - wn^2 + \delta\Delta(w - \beta n).$$

Then, at that point, by linear analysis for the three equilibrium points of framework (1), Hopf bifurcation and Turing bifurcation can be obtained. As an increases or β diminishes, standard examples will disappear and become homogeneous distribution or uncovered state. To exhaustively investigate the impact of soil-water diffusion criticism on designs formation, we changed the degree of input intensity by altering the worth of β. It is tracked down that alteration of the criticism intensity β can cause design transition. As the boundary increases from little to enormous, designs change as following arrangement: whole designs, stripe examples and spot designs. At the point when the input intensity of soil moisture diffusion is sufficiently huge, plants need a lot of water to survive, and the competition among plants is fiercer. This will prompt the passing of certain plants. The criticism intensity with a sufficiently enormous worth can cause the disappearance of vegetation or vegetation designs. In addition, plant normal density is positively corresponded with the soil-water diffusion criticism, which is a positive criticism in Turing domain. Their outcomes uncover the relationship between criticism intensity and vegetation design dynamic. Be that as it may, because of the complexity of vegetation environments, there is no sufficient experiment to help this soil-water diffusion input mechanism.

The impacts of rainfall, plant misfortunes and delicate incline on the distribution of vegetation have been studied previously. The inhomogeneous distribution of these variables in the environment is the main justification behind vegetation designs. Plants themselves additionally can induce heterogeneity of soil, for example, nearby consumption of soil nutrients, soil microorganisms and microbes and water absorption. Plant-soil negative criticism is an overall designation for a portion of those mechanism, which induce unfavorable survival conditions

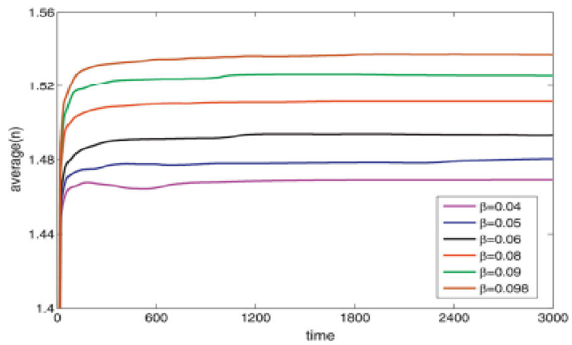

Figure 1: Average vegetation density at various feedback strengths (β).

for conspecific plants. Previous examination on this viewpoint seldom involved the interaction between plant development structures and PSNF on the rise of vegetation designs. Typically, Vincenot in presented a spatially explicit hybrid model to investigate the relation. This hybrid model describes plant metabolism, seed dispersal, stochastic age-subordinate passing and toxic dynamics in soil, in which toxic dynamics represents a theoretical course of species-specific autotoxic compounds. The following equation depicts the dynamics of plant biomass:

$$\frac{dB}{dt} = gB\left(1 - \frac{B}{K}\right) - (d + sT)B,$$

Scale-dependent mechanisms

Scale-subordinate mechanisms generally mean positive criticism and negative input occurring at different spatial scales. In biological systems, it can be concretized as brief distance facilitation and significant distance inhibition, rescinding from precise mechanisms involved. This mechanism especially provides a device to explain the examples from frameworks with little potential environmental heterogeneity. In youthful mussel beds, they structure aggregations that make mussels and sediment shows a stunned distribution; therein, sediment contains no mussels. In the high-density patches where mussels accumulate, mussels bind to one another using byssus strings then areas of strength for structure. Consequently, aggregation can decrease mussel misfortunes owing to wave disturbance and predation by its predators. Notwithstanding, high density of mussels likewise increases depletion of available algal food. Subsequently, aggregation can intensify competition among mussels and afterward seriously diminish mussel development. It ought to be noticed that green growth assets consumed by mussels are carried by tidal flows over significant distances; subsequently, competition for algal produces results over a lot bigger distances. Owing to those findings, Koppel et al. proposed and afterward demonstrated a hypothesis that scale-subordinate criticism can induce the noticed examples in mussel banks. All the more specifically, short-range facilitation by common protection from waves and flows and long-range competition for green growth contribute to design formation.

Conclusion

Design formation of ecological frameworks is an important issue of normal worry by mathematician, biologist, ecologist and government staff. Thus, we show a review on spatial examples of ecological frameworks from the part of mathematical modeling. We start from two realistic ecological frameworks (mussel and vegetation) and uncover that different sorts of mechanisms might prompt the rise of examples. This function admirably links dynamical modeling to ecological functions in ecological frameworks. It ought to be noticed that we simply show single mechanism of example formation including criticism, scale dependent, stage separation, nonlocal impacts, time delay and spatial heterogeneity in this work. In any case, if a few elements (for instance, criticism and time defer in hematopoietic immature microorganisms dynamics work simultaneously, then, at that point, we want to answer that what the dominant variable of examples formation are and the way in which they interact with one another. The question should be tended to by coupling these impacts with genuine information.

Reference

1. van de Koppel, J., Rietkerk, M., Dankers, N., et al.: Scaledependent feedback and regular spatial patterns in young mussel beds. Am. Nat. 165, E66–E77 (2005)
2. Binney, H., Edwards, M., Macias-Fauria, M., et al.: Vegetation of Eurasia from the last glacial maximum to present: Key biogeographic patterns. Quat. Sci. Rev. 157, 80–97 (2017)
3. Kondo, S., Miura, T.: Reaction-diffusion model as a framework for understanding biological pattern formation. Science 329, 1616–1620 (2010)
4. Alon, U.: An Introduction to Systems Biology. Chapman & Hall/CRC, Boca Raton, FL (2006)
5. Murray, J.D.: Mathematical Biology II: Spatial Models and Biomedical Applications. Springer, New York (2003)

6. Fuentes, M., Kuperman, M., Kenkre, V.: Nonlocal interaction effects on pattern formation in population dynamics. Phys. Rev. Lett. 91, 158104 (2003)
7. Kenkre, V.M., Lindenberg, K.: Modern challenges in statistical mechanics: patterns, noise, and the interplay of nonlinearity and complexity. In: Modern Challenges in Statistical Mechanics: Patterns, Noise, and the Interplay of Nonlinearity and Complexity (2003)
8. Fuentes, M.A., Kuperman, M.N., Kenkre, V.M.: Analytical considerations in the study of spatial patterns arising from nonlocal interaction effects. J. Phys. Chem. B 108, 10505– 10508 (2004)
9. Liu, Q.-X., Doelman, A., Rottschäfer, V., et al.: Phase separation explains a new class of self-organized spatial patterns in ecological systems. Proc. Natl. Acad. Sci. 110, 11905– 11910 (2013)
10. Klausmeier, C.A.: Regular and irregular patterns in semiarid vegetation. Science 284, 1826–1828 (1999)
11. Ni, W., Shi, J., Wang, M.: Global stability and pattern formation in a nonlocal diffusive Lotka-Volterra competition model. J. Differ. Equ. 264, 6891–6932 (2018)
12. Tian, C., Ling, Z., Zhang, L.: Nonlocal interaction driven pattern formation in a prey-predator model. Appl. Math. Comput. 308, 73–83 (2017)
13. Turing, A.M.: The chemical basis of morphogenesis. Philos. Trans. R. Soc. Lond. B 237, 37–72 (1952)
14. Koch, A., Meinhardt, H.: Biological pattern formation: from basic mechanisms to complex structures. Rev. Mod. Phys. 66, 1481 (1994)
15. Jacob, F., Monod, J.: Genetic regulatory mechanisms in the synthesis of proteins. J. Mol. Biol. 3, 318–356 (1961)
16. Holling, C.S.: Resilience and stability of ecological systems. Ann. Rev. Ecol. Syst. 4, 1–23 (1973)
17. Meron, E.: Modeling dryland landscapes. Math. Model. Nat. Phenom. 6, 163–187 (2011)
18. Sun, G.-Q., Wang, C.-H., Chang, L.-L., et al.: Effects of feedback regulation on vegetation patterns in semi-arid environments. Appl. Math. Model. 61, 200–215 (2018)
19. von Hardenberg, J., Meron, E., Shachak, M., et al.: Diversity of vegetation patterns and desertification. Phys. Rev. Lett. 87, 198101 (2001)
20. Wedin, D., Tilman, D.: Competition among grasses along a nitrogen gradient: initial conditions and mechanisms of competition. Ecol. Monogr. 63, 199–229 (1993)